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Cost-Optimized NPK Fertilizer Recommendation Using Mixed Integer Linear Programming

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Abstract.
Fertilization is a major cost in soybean production, and inefficient fertilizer selection, combined with the indivisibility of commercial packaging, increases expenses. This study developed a decision-support system to recommend the lowest-cost combination of NPK fertilizers using Mixed Integer Linear Programming (MILP). The system includes an Android client and a Python (FastAPI) backend with PuLP and the CBC solver, using soil analysis in accordance with Paraná fertilization guidelines. It provides a ranking of solutions via a cardinality constraint and supports both uniform and management-zone (MZ) modes. In a 15.5 ha case study in Paraná, restricting the catalog to single-nutrient fertilizers increased costs by 11% to 46%. At the same time, the MZ mode identified localized potassium depletion not captured by the uniform approach. All results matched manual calculations, confirming the model's validity.

Keywords.
precision agriculture; soybean; combinatorial optimization

Introduction

NPK fertilizers are essential to crop productivity, but fertilization is among the costliest operations, and generic application that disregards soil analysis can lead to waste and nutrient imbalance. Studies have used linear programming (LP) to find least-cost fertilizer blends (Tieppo et al., 2011), and mixed-integer linear programming (MILP) has been applied to fertilizer purchasing for soybean (Vanzela et al., 2020). It is essential to consider the indivisibility of commercial packaging, as fertilizers are sold in closed bags whose weights must be respected. Thus, we developed a computational tool to recommend the least-cost NPK combinations available in western Paraná, respecting packaging indivisibility, for both uniform and management zone (MZ) fertilization, and to quantify the cost effects of catalog composition and MZ granularity.

Materials and Methods

The recommendation was modeled as a least-cost blending problem. Let F be the set of fertilizers, each source i having a unit cost c and guaranteed contents α of N, P_2O_5 and K_2O; the variable x

denoted the mass applied. The objective minimized the total acquisition cost (Eq. 1), subject to meeting the demand of each nutrient (Eq. 2).

$$Z = \sum_{i \in F} c_i x_i \tag{1}$$

$$\sum_{i \in F} \alpha_{ij} x_i \geq R_j, \quad j \in \{P, K\} \tag{2}$$

In the closed-package mode, x_i was an integer multiple of the package weight W ($x_i = W * n_i$, where n_i is a non-negative integer), turning the model into an MILP solved by the branch-and-cut algorithm. A cardinality constraint $\Sigma\, y_i \leq K_{max}$, with a binary variable y_i linked to x_i via a big-M constraint, limited the number of distinct sources; the model was solved with K_{max} set to 3, 2, and 1, producing a ranking ordered by logistical simplicity. The required N was set to zero, as soybeans fix nitrogen biologically.

An Android client (Kotlin/Jetpack Compose) and a Python backend (FastAPI) built the model with PuLP and solved it with CBC. Soil P and K were classified and converted to P_2O_5 and K_2O doses according to the manual for Paraná (Pauletti & Motta, 2019). The system was validated on a 15.5 ha field in Céu Azul, Paraná, Brazil, using 40 georeferenced sampling points from 2013 and 2017 and MZs (divided into 2, 3, and 4 classes). Soybean was the reference crop (3.5 t ha^{-1}), with a 23-product catalog from six suppliers and three scenarios (all, single-nutrient, multi-nutrient). All outputs were verified against independent manual calculations.

Results and Discussion

In both years, P and K were classified as Very High in nearly all zones, resulting in maintenance doses of 40 kg ha^{-1} of P_2O_5 and K_2O. The mixed fertilizer 00-23-21 (big bag) was the dominant source, supplying both nutrients at the lowest unit cost. With the full catalog, the average-field recommendation cost was R$ 6,40, compared to R$ 6,67 to R$ 8,67 for the MZ configurations (2 to 4 zones). Restricting the catalog to single-nutrient fertilizers increased the lowest-cost recommendation by 11% to 46%, as phosphorus and potassium then required separate products and smaller packages, increasing logistical costs; the multi-nutrient scenario nearly matched the full catalog. The cardinality ranking quantified the cost of simplification: reducing from four to three sources increased the cost by only about 2%, whereas reducing to two sources raised the cost by approximately 20%, and the single-source solution was infeasible with single-nutrient products, since none could supply both P and K simultaneously. The MZ mode did not always reduce costs, as packaging indivisibility required at least one big bag per zone and the field was relatively homogeneous; nevertheless, it identified localized potassium depletion in 2017 (one sub-region reclassified from Very High to High) that the uniform approach could not capture. All system outputs matched the manual calculations exactly, validating the implementation.

Conclusion

The MILP-based system has recommended the least-cost NPK fertilizer combinations for soybean while respecting packaging indivisibility, in both uniform and MZ modes. The optimal cost depended on the catalog and on MZ granularity, both quantified through a cardinality-based ranking: restricting to single-nutrient fertilizers raised costs by up to 46%, and the zone mode revealed localized potassium depletion missed by the uniform approach.

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